Pool evaporation under low Grashof number downward convection

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ABSTRACT

We investigate the free evaporation of water into air from a small-scale (~0.1 m) circular pool for low Grashof (Gr) number values under dominant downward flow motion. Repeatable experiments performed at atmospheric pressure for air temperatures at 290 K, 300 K and 310 K and relative humidity values ranging from 30% to 60%, are described and compared to detailed Finite Element Method (FEM) numerical simulations. For the free evaporation regimes considered here, a downward thermally-induced flow originated at the rim of the pool overcomes the concentration-induced buoyancy, consequently forcing the far-stream warm dry air to descend into the lower temperature air-water interface. Experimental results show that a steady-state recirculation zone near the air-water interface develops for Gr numbers greater than or equal to 50. The Sherwood number (Sh) for this geometry appropriately scales with Gr for drier free-stream boundary conditions, while it approaches a constant value for smaller mass transfer potentials (i.e., more humid environments). A stable Sherwood number (Sh) correlation as a function of Gr values is proposed, covering Gr values from 10 to 10^5.

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1. Introduction

Natural convection driven by the evaporation of water into moist air is an intricate multi-physics process involving combined heat and mass transfer. The Grashof (Gr) numbers relevant to laboratory-scale (0.1 to 1 m pool dimensions) evaporation of water in air under gravitational forcing range from unity to several million, where the small Gr number processes are unstable due to the intermittent formation of toroidal ring-like jets and recirculation zones near the air-water interface. As such, detailed rates of water evaporation into air are needed to understand a number of natural processes and engineered applications such as spore dispersal mechanisms developed by fungi [1], water desalination systems [2], large scale geothermal engineering [3,4] and microscopic DNA mapping [5,6]. Several recent papers investigate geometrical, composition, and surface related effects on gravitationally-driven liquid evaporation under different environmental conditions (see, e.g., [7–11]).

While there are many experimental and theoretical results for the convection driven by the evaporation from a surface of liquid water into quiescent air, most of the correlations available correspond to relatively large Gr (or Rayleigh, Ra) numbers. The high Gr numbers usually result from: (1) the addition of external thermal energy to the liquid side of the interface; or (2) a length scale of the pool of the order of 1 m or larger. At high Gr numbers, opposing effects from thermal and concentration potentials, as well the effects of incipient recirculations zones, are negligible. Consequently, the existing literature contains significantly fewer studies at lower Grashof numbers where at least one of the previous conditions does not apply, see [12,13].

In this work, we study Gr number regimes relevant to the common phenomena involving pool sizes comparable to a glass of water or a laboratory beaker, where opposing buoyant effect and recirculation zones above the air-water interface are present. We develop precision-controlled experiments and appropriate numerical simulations to study stable conditions that naturally form from the thermally-driven downward convection towards the interface. For Grashof numbers on the order of 50 and above, experimental results reveal that the recirculation zone is stable, that is, moderate heterogeneities of the surrounding impart a negligible effect in the net mass and heat transfer. Numerical simulations then extended to lower Grashof numbers indicate that the Sherwood number converges to a nominal value (the pure diffusion limit). In the light of this result, we propose a universal correlation valid for a range of Gr numbers between 10 and 100,000.

As mentioned by Sharpley, Boelter, Gordon, and Griffin [14,15], small water vapor concentration differences results in substantial changes in the flow structure above the air-water interface. When the concentration difference is relatively high (e.g., heat supply at the liquid side) the flow is directed primarily upward (also referred to as parallel-flow evaporation), that is, a rising plume will form...
above the interface with the saturated air leaving in an upward jet structure. As the concentration difference decreases (e.g. free evaporation), the mixture above the air-water interface is denser, leading to a dominant downward flow, also known as contraflow condition. To interpret these two distinct flow configurations, a common approach is to assume that a boundary layer forms at the interface, while the temperature of the liquid remains fixed at some predefined value [16]. The dry air enters the boundary layer from the edges of the container when the temperature of the water is higher than the ambient temperature. In this case, the incoming air is characterized by a quasi-stagnant velocity and enters the boundary layer at near ambient temperatures. If instead, the surface water is colder (not so cold as to induce condensation), the dry air will enter the boundary layer at the center of the interface.

Several studies on water evaporation from a heated liquid pool have been conducted. The heated pool induces upward convection above the air-water interface when the interface is at a higher temperature than the surrounding air-vapor mixture. Sharpley and Boelter [14], provided some of the earlier results suggesting a correlation of the form $Sh = C Sc^{1/3} Ra^{n}$, where $C$ is constant, to estimate the evaporation rate from heated pans in quiescent air. These earlier correlations were based solely on experimental observations. Similar studies were later conducted by Boelter et al. [15], Sparrow et al. [17] and more recently by Bower and Saylor [12]. Bower and Saylor [12,13] concluded that for an upward convection (processes where $T_i > T_0$) the power $n$ is affected by turbulence and restrictions of the flow in the proximity of the air-water interface. For example, power $n$ values found from experimental investigations ranged from 0.213 to 0.306. Bower and Saylor [12,13] provided a summary of previous work in this area, including a compilation of correlations with applicable ranges (see Table 1 of their work). Dollet and Boulogne [18] also studied evaporation under upward convection, but for an isothermally evaporating disk, with the disk geometry defined as a circular evaporating surface laying on an infinite plane. In their work, the authors derive a $Sh$ number correlation based on the flux from different zones that theoretically develop in the boundary layer above the air-water interface. They suggest a correlation where $Sh_{\text{R}} \sim \pi/2 (C_1 + C_2 Gr_{\text{R}}^{1/2})$ with constants $C_1$ and $C_2$ obtained from experimental results.

Among the $Sh$ number correlations, Sparrow et al. [17] is the most relevant for evaporation driven by downward flow, a flow structure present in processes where the air-water interface is colder than the surrounding. The low-temperature interface results in opposing thermal-concentration buoyant effects - the central point of this study. Engineering applications based on the free evaporation of water where the evaporative cooling effect is of central interest [19–21] have emerged recently. Because of the lack of work investigating processes under downward convection, we focus this study on the evaporation under downward convection on small (centimeter) scale pool containers where the body of water is generally not heated.

Although a boundary layer interpretation provides valuable insights, the approach is not generally valid when small scale evaporation processes occur in the presence of opposing thermal-concentration buoyant flows due to inexistence of a length scale that is much smaller than the integral length scale. Opposing thermal-concentration effects may induce the formation of recirculation zones, which render the analysis of the flow by analytical methods substantially more challenging. In buoyant flows originating from a flat horizontal surface, the direction of buoyancy is often orthogonal to the horizontal surface, and for fluids characterized by $Sc = Pr (Le \neq 1)$ the boundary layer approximation is not applicable due to development of stagnation regions with associated recirculation zones near the surface [22,23]. These stable but weak flow fields create additional challenges for experimental observation at such low Grashof number flows, ultimately limit-
ing the range of available correlations for use in laboratory-size or tabletop-size evaporative pools.

In addition to carefully designed experiments, we use Finite Element Model (FEM) numerical simulations to simulate the natural convection evaporative processes in the presence of downward thermal and upward concentration buoyancy induced flows. The simulations are carried on without the need to enforce a predefined temperature at the liquid phase interface, thus reproducing the experimental setup. As such, we allow evaporative cooling to take place so that there is equilibrium at the air-water interface for given environmental temperatures and relative humidities. Boundary conditions at the interface are defined in accordance to heat and mass balances such that the heat of vaporization is the sum of the diffusive-convective heat flow contributions from the liquid side, from the gas side and from surface radiation. Because mass fractions of water vapor in air are small at room temperatures (this is a low mass transfer process), the downward thermal-induced buoyant force dominates over the concentration-induced buoyancy, which results in a net downward oriented flow in the far-stream field, sustained by a toroidal ring-like jet originating at the rim of the pool.

Here we minimize constraints imposed on the air-water interface to allow for a naturally achieved equilibrium of the system given the far-stream e-state condition. This is done to emulate the general situation of free evaporation processes where the thermal condition of the liquid pool is established by the thermal balance at the interface. The same is true for the numerical simulations where the condition at the interface is given according to the aforementioned energy balances at very low Gr numbers that are characteristic of water evaporation in air at room temperatures.

2. The physical problem

Water evaporates from a cylindrical container in a controlled environment. Constant values of temperature, pressure and relative humidity away from the air-water interface define both the thermodynamic condition and the water vapor concentration of the (far-stream) environment, or the e-state (see Fig. 1). Under quasi-steady conditions, the evaporation process achieves a near-stationary rate for each combination of T, P, and RH. Gravity-induced natural convection due to thermal and concentration gradients are the main driving mechanisms for fluid motion on the gas side. For modeling purposes, we assume the buoyant forces to follow the Boussinesq approximation, where the density variations are considered to be relevant only in the buoyancy terms of the momentum equation. The radius of the container is the length scale of interest for both the thermal and mass Grashof numbers:

\[
 Gr_h = \frac{g\beta_h(T - T_e)R^3}{\nu_h^2}
\]

(1a)

\[
 Gr_m = \frac{g\beta_m(m_{1.5} - m_{1.6})R^3}{\nu_m^2}
\]

(1b)

as well as for the Sherwood number:

\[
 Sh_h = \frac{\frac{1}{\beta_h} R}{\rho \nu_h^{\frac{1}{2}}} = \frac{\bar{m}^\prime R}{\bar{m}_m R_{12}}.
\]

(2)

The interface mass flux \( \bar{m}^\prime \) and the mass driving potential \( \bar{m}_m \) are related through the mass transfer conductance of species 1 (water vapor) \( \bar{m}^\prime = \bar{m}_m \bar{m}_1 \), while the mass driving potential is defined as (for more details see reference [24])

\[
 \bar{m}_m = \frac{m_{1.5} - m_{1.6}}{1 - m_{1.5}}.
\]

(3)

The average Sherwood number is expressed as a function of the combined thermal and concentration Grashof number, and averaged over the entire surface area of the air-water interface s.

The heat flux from the liquid side, from the gas side, and the radiative heat exchange between the air-water interface and the surrounding environment all play a part in determining the combined heat and mass transfer rates for given conditions. The sensible, latent and radiative heat fluxes are all very small in this problem, so the relative balance between each of these different transfer mechanisms requires careful consideration. Here we assume an infinitely thin interface without impurities, which leads to temperature continuity (i.e., thermal equilibrium) on both sides of the air-water interface. The interface temperature drops due to the loss of thermal energy from the liquid molecules left in the liquid state as water molecules evaporate into the vapor phase. Due to the high enthalpy of vaporization of water, the temperature drop at the interface is felt throughout the depth of the liquid pool. The liquid pool is shallow for its characteristic conduction length scale, therefore convection and surface tension circulations induced by small thermal and density gradients on the liquid side are weak, so that the heat flux from the liquid side to the interface approaches the
value found by considering a hydrostatic pool with no circulation induced by thermal effects. However, the nonslip condition at the air-water interface induces liquid motion, and this effect is taken into account in our numerical simulations. Under normal conditions of temperature and pressure, the equilibrium saturation temperature (approximately the wet bulb temperature) and associated mass fraction of water vapor on the gas side are both too small to induce Stefan flow across the interface, so the problem under study here involves low mass transfer rates only.

Overall, the low mass transfer rate process is a combined heat and mass transfer problem regulated by a delicate balance between natural convection and radiative heat transfer rates. The convective transfer process results from complex gravity-induced flow patterns on the gas side of the interface.

2.1. Governing equations and boundary conditions

On the liquid side, the governing equations for mass, momentum, and energy conservation are:

\[ \nabla \cdot \mathbf{u} = 0 \]  \hspace{1cm} (4)

\[ \mathbf{u} \cdot \nabla \mathbf{u} = - \frac{1}{\rho} \nabla p + v_0 \nabla^2 \mathbf{u}, \]  \hspace{1cm} (5)

\[ \mathbf{u} \cdot \nabla T = \alpha_T \nabla^2 T. \]  \hspace{1cm} (6)

The governing equations on the gas side are also derived from the mass, momentum, energy and species conservation equations. The gas is considered a binary ideal gas system formed by water vapor and dry air:

\[ \nabla \cdot \mathbf{u} = 0, \]  \hspace{1cm} (7)

\[ \mathbf{u} \cdot \nabla \mathbf{u} = - \frac{1}{\rho_g} \nabla p + v_0 \nabla^2 \mathbf{u} + g(\beta_m \Delta m_1 + \beta_T \Delta T) \hat{k}, \]  \hspace{1cm} (8)

\[ \mathbf{u} \cdot \nabla T = \alpha_g \nabla^2 T \]  \hspace{1cm} (9)

\[ \mathbf{u} \cdot \nabla m_1 = \dot{\mathcal{D}}_{12} \nabla^2 m_1. \]  \hspace{1cm} (10)

The symbol \( \dot{\mathcal{D}} \) indicates pressure (including the gravitational potential \( p + gz \rho_g \)), and the symbols \( \beta_m \) and \( \beta_T \) are respectively the mass and thermal volumetric coefficients of expansion. More details about these coefficients are given in Appendix A. Since the absorption of air in water is negligible, we assume that only one species is being transferred (water vapor and denoted by the subscript 1) at the interface.

The set of governing equations are subjected to the following boundary conditions (see Fig. 1.a for reference). At all surfaces at the e-state we have

\[ \mathbf{u} = 0 \]  \hspace{1cm} (11a)

\[ T = T_e \]  \hspace{1cm} (11b)

\[ m_1 = m_1(RH_e, T_e) \]  \hspace{1cm} (11c)

\[ \forall \ x, y, z \ in \ e. \]

At the air-water interface (using the Cartesian coordinate system depicted in Fig. 1):

\[ u_x(z = 0^+) = u_x(z = 0^-) \]  \hspace{1cm} (12a)

\[ u_y(z = 0^+) = u_y(z = 0^-) \]  \hspace{1cm} (12b)

\[ u_z = 0 \]  \hspace{1cm} (12c)

\[ \mu_g \frac{\partial u_x}{\partial z} \bigg|_{z=0^+} = \mu_l \frac{\partial u_x}{\partial z} \bigg|_{z=0^-} \]  \hspace{1cm} (12d)

\[ \mu_g \frac{\partial u_y}{\partial z} \bigg|_{z=0^+} = \mu_l \frac{\partial u_y}{\partial z} \bigg|_{z=0^-} \]  \hspace{1cm} (12e)

\[ T(z = 0^-) = T(z = 0^+) = T_i(r) \]  \hspace{1cm} (12f)

\[ -\rho_g \dot{\mathcal{D}}_{12} \frac{\partial m_1}{\partial z} \bigg|_{z=0^+} = h_g + \epsilon \sigma (T_i^4 - T_e^4) \]  \hspace{1cm} (12g)

\[ -k_g \frac{\partial T}{\partial z} \bigg|_{z=0^+} - k_l \frac{\partial T}{\partial z} \bigg|_{z=0^-} . \]

\[ m_1 = m_1(P_{sat}, T_i) \]  \hspace{1cm} (12h)

\[ \forall \ x, y \ in \ s, \ and \ for \ z \ also \ in \ s \ (z = 0). \]  

The symbol \( \overline{\rho}_g \) refers to the binary diffusion coefficient of water vapor in air [24] (see appendix). \( h_g \) is the enthalpy of vaporization evaluated at \( T_e \), and \( \epsilon \) to the emissivity of the water, here considered as a blackbody for longwave radiation. Note that the interface energy balance (Eq. 12g) implies low mass transfer rates explicitly, as expected for a air-water interface at ambient temperatures. Inclusion of Stefan fluxes does not change the numerical results for the conditions under study.

The water vapor at the interface is saturated at the local inter- face temperature, \( T_i \). The mass fraction \( m_1 \) depends on the partial pressure of species 1, which in turn depends on the saturation pressure. The saturation pressure for water vapor is obtained according to the Tetens equation of state [25], as described in the Appendix.

Continuity of temperature and continuity of heat flow are imposed between the vertical walls of the container and the surrounding fluid (water or air), while zero mass flux is applied at the walls of the container exposed to the gas side:

\[ T = T_i \]  \hspace{1cm} (13a)

\[ \mathbf{n} \cdot g = k_l \nabla T \bigg|_{r} \]  \hspace{1cm} (13b)

\[ \mathbf{n} \cdot m_1 = 0 \]  \hspace{1cm} (13c)

\[ \forall \ x \ in \ w, \ \forall \ y \ in \ w, \ and \ \forall \ z \ in \ w. \]

In order to investigate the effect of boundary conditions at the bottom of the liquid pool, we consider two different conditions: specified temperature equal to \( T_e \) (equivalent to assuming a thermal reservoir at temperature \( T_e \) under the container); and zero heat flow at the bottom of the pool. The real condition depends on the thermal losses by the pool walls. In this study, the dimensions of the pool were selected to minimize surface tension (meniscus) and internal circulation effects.

3. Mass transfer correlations

The Sherwood number \( Sh \) for forced flow evaporation can be expressed as \( Sh = Sh_0 + C \alpha m e^n \), so that

\[ Sh = Sh_0 + C \left( \frac{UL_e}{v} \right)^m \left( \frac{v}{\dot{\mathcal{D}}_{12}} \right)^n \]  \hspace{1cm} (14)

in which \( Sh_0 \) is the dimensionless mass transfer rate in the pure diffusion limit, \( U \) the characteristic velocity and \( L_e \) the characteristic length. The exponent \( m \) is 1/2 for laminar flow and approaches 4/5 for turbulent flow, whereas the exponent \( n \) is 1/3 for Pr numbers higher than 0.5 in laminar flows.

In free evaporation, the characteristic velocity of the flow depends on the ambient condition into which the process is subjected. In a dry atmosphere, for instance, the velocity of the gas
reaches values on the order of $10^{-2}$ m/s. Since in this problem the characteristic length is on the order of centimeters, the Grashof number is of order 10, indicating that the inertial forces are slightly dominant over the viscous forces. Under this circumstance, the buoyancy term in the conservation of momentum equation scales with the convective term, resulting in a characteristic velocity proportional to the square root of the total Grashof number:

$$\mathcal{O}\left(\frac{U^2}{L}\right) \sim \mathcal{O}(g\beta_1(T - T_e) + g\beta_m(m - m_1, e)).$$

(15a)

or

$$U \sim \frac{V}{L} \sqrt{Gr L}. \quad (15b)$$

For evaporation occurring at high relative humidity, the velocity of the system tends to zero due to the weaker driving forces, in turn leading to a buoyant force proportional to the viscous forces and the characteristic velocity proportional to the total Grashof number:

$$\mathcal{O}\left(\frac{V U}{L^2}\right) \sim \mathcal{O}(g\beta_1(T - T_e) + g\beta_m(m - m_1, e)).$$

(16a)

$$U \sim \frac{V}{L} \sqrt{Gr L}. \quad (16b)$$

The Sherwood number would then become proportional to $Gr^{1/2}$. However, the temperature and mass concentration gradients are greatly reduced in this condition, which lead to variations in the Sherwood number likely negligible for Grashof numbers near zero. As a result, for Grashof numbers near zero the dimensionless evaporation rate would be around the diffusion limit value, that is

$$Sh = Sh_0 + C_s Sc^{1/3} Gr^{1/2} \approx Sh_0$$

(17a)

to then increase proportionally to the 1/4 power on the Grashof, as the Grashof number increases:

$$Sh = C_b + C_e Sc^{1/3} Gr^{1/4}. \quad (17b)$$

4. Experimental method

A small container is completely filled with water and left to evaporate inside a vacuum-sealed chamber, as shown in Fig. 1. The atmosphere of the chamber is thermodynamically controlled and the container is placed over a high precision balance (4212B-102, A&D) that measures the remaining amount of liquid water. The chamber, made of stainless steel 304L, has a cylindrical shape with the inner diameter and inner length respectively of 0.3 m and 0.7 m in size. The longitudinal axis of the chamber is in the horizontal position, with the balance placed in the center as showed by Fig. 1. The chamber was manufactured to have channels along the walls serving as passages for a working fluid driven by a chiller (Thermo Scientific Accel 500 LT). Through the recirculation of the working fluid it is possible to set the temperature of the walls of the chamber to a desired value.

The chamber is properly equipped with sensors that allow the full control over its thermodynamic state. A high precision chilled mirror dewpoint hygrometer (Optidew Remote, Michell) is used to measure the relative humidity of the chamber's interior. The relative humidity is decreased by forcing the air against a compartment filled with molecular sieve, which retains the water vapor molecules in its pores while drying out the air. Ten type K thermocouples are used to measure the temperature inside the chamber, 6 of which are directly attached to the walls. The inside pressure can be set to any value within the range of 1 to 0.1 atm. Once defined, the pressure at which the experiment is conducted is kept constant while a capacitance manometer (CMX1, Brooks Instrument) records the pressure level. Lastly, the temperature of the air-water interface is measured by an infrared sensor (1045 B, Phidgets) with a $10^5$ field of view. The sensor is positioned in such a way as to read only the average temperature associated with the interface.

Fig. 2 is a schematic depiction of the experimental setup.

For each experimental run, the balance is re-calibrated and the hygrometer reset. In order to assure that the system is in quasi-steady state, the first one to two hours after the preparation stage are discarded. As the water evaporates, the level of water in the container decreases, and to guarantee that the container stays completely filled during the measurements, the entire process is repeated for different targets of relative humidity. No mechanism keeping the relative humidity constant is applied during the experiments, in an effort to avoid disturbances influencing the system and, as consequence, the relative humidity continuously increases as the experiment goes on. However, owed to the fact that natural evaporation is a slow process, the same experiment can still be considered in quasi-steady state if it is divided into sub-experiments with sufficiently small relative humidity variation. Here, the sub-experiment is a time window in which the difference between the maximum and minimum relative humidity values is less than 1%.

After the experimental run finishes, a series of mass fluxes are obtained by performing a linear regression on the acquired mass and time data belonging to the selected time window (or to the sub-experiment, as explained previously). A $\chi^2$ test is used to determine the number of degrees of freedom (DOF - number of data points minus the number of parameters in the fit) to be used, in which the number of DOF is progressively increased until it includes all the values in the respective window, starting with a minimum of 200 DOF. The mass flux is obtained from the data range corresponding to a number DOF yielding the best goodness of fit. All the other measured variables are calculated as the average of all the values belonging to the selected range.

5. Numerical method

A three-dimensional geometry, composed by the gas and liquid domains is solved using COMSOL Multiphysics™, which employs the finite element method for the solution of boundary value problems. We implement the governing equations along with the boundary conditions in a steady-state simulation using a direct solver, and apply a relative residual tolerance of $10^{-3}$. To account for the cooling effect due to the phase change of water, we consider the interface as a boundary heat source in which energy is lost through heat of vaporization and energy is added through radiative heat exchange. The sum of all heat fluxes is compared to the total heat of vaporization for every numerical simulation, as a means to check the fidelity of the energy balance (Eq. 12g). All differences found were smaller than 0.05%. The water container is modeled as a shell since the thickness of the water container is considerably smaller than the characteristic length of the chamber (<1 mm vs. 300 mm). We enforce continuity of temperature and continuity of heat flux between the walls and the surrounding fluid. In order to employ fluxes as boundary conditions, the method of Langrange multipliers was used instead of directly taking the local gradient of the variables in question.

The geometry is discretized using tetrahedral elements, including a thorough mesh refinement near the air-water interface. Results between nodes in the mesh are interpolated using cubic shape functions. A mesh independence study is reported in Table 1, with the Sherwood number given in terms of the number of triangles at the interface as well as in terms of the total degrees of freedom. One can notice that using more than approximately 1000 triangle elements at the interface results in minor changes of the Sherwood number.
6. Results

Here we show the central result of this study, a $Sh$-correlation valid for Grashof numbers in a range of $10$ to $10^5$. The correlation is obtained from numerical results in excellent agreement with experimental data, covering a wide range of temperature and relative humidity conditions. Overall, the correlation recovers the pure diffusion limit as the Grashof number goes to zero and obeys the scaling law followed by unbounded free evaporation.

6.1. Evaporation in the purely diffusive limit

In the purely diffusive limit, the velocity of the system is zero and both the temperature and the species concentration fields satisfy the Laplace equation. Boundary conditions are given by Eqs. 11, 12 (with all velocities equal to zero) and 13. The problem of free evaporation from a pool at the purely diffusive limit is analogous to the electrostatic problem of two oppositely charged circular parallel conducting disks of radius $a$ separated by a distance $ka$. The potential field in this case is made of plane-symmetric spheroids superposing each other [26,27]. In the case of free evaporation from a pool with no convection, numerical results also indicate that the water vapor concentration profiles are plane-symmetric spheroids (see Fig. 3) with the plane of symmetry defined by the plane supporting the pool. Far away from the air-water interface, the water vapor concentration remains at a predefined value according to the far-stream relative humidity condition. The evaporation rate depends weakly on how far the air-water interface is from the plane of symmetry, that is, the height of the pool - an

![Fig. 2. Schematic view of the vacuum-sealed chamber (a) and its interior (b, section A-A), the experimental setup used to measure the evaporation rate of water from a small container under downward convection. The high precision balance weights the remaining liquid water inside the pool as water evaporates. A manometer positioned away from the pool records the relative humidity inside the chamber. A manometer positioned on top of the setup reads the interior pressure, and 10 thermocouples type-K (not shown in the figure) spread throughout read the chamber's interior and wall temperatures. A downward-facing infrared (IR) sensor measures the averaged air-water interface temperature.](image)

![Fig. 3. Normalized water vapor mass concentration $\phi$ for water evaporating into air from a small pool in the pure diffusion limit. The results are obtained from numerical simulations, according to three different $HR$ ratios. The water vapor concentration isosurfaces approach asymmetric oblate spheroidal shapes. When the container is short, the concentration isosurfaces near the container's walls are confined to a small region, yielding maximum evaporation rates. The isosurfaces spread apart as the container height is increased, until a point after which increasing the container's height has no significant effect on evaporation rates. See Table 2 for complementary information.](image)
Table 2
Sherwood number values for the evaporation from a small pool in the purely diffusive limit as a function of the ratio HR.

<table>
<thead>
<tr>
<th>HR</th>
<th>1/3</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sh</td>
<td>2.4</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.9</td>
</tr>
</tbody>
</table>

analogous interpretation to the charged disks problem where decreasing the separation between plates increases the influence of charge distribution from one plate to the other [27].

One difference between the evaporation from a pool under negligible convection and the oppositely charged parallel disks is that, in the former, there is a zero flux boundary condition at the walls of the container, while in the latter such condition is nonexistent. Fig. 3 shows the normalized mass concentration \( \phi \) for three different heights, while Table 2 contains the respective Sherwood number values. In short containers, the distribution of the isolines of mass concentration near the walls is confined into a small space, forcing slightly larger mass fluxes. As the height of the pool increases, the isolines of water vapor become gradually less compressed, until a point in which increasing further the height produces no significant effect.

Under the idealized scenario of a purely diffusive regime, the Sh number shows a weak dependency on the height of the pool, as demonstrated by the results of numerical simulations in the pure diffusion limit (Table 2). Since such dependence is weak, it is reasonable to expect that the Sh number (based on the radius of the pool) asymptotes to values around 2 when the Gr number approaches zero for a wide range of pool heights.

6.2. Evaporation under downward flow motion

Fig. 4 shows the mass flux along with the temperature of the interface as a function of relative humidity for different ambient conditions, and for the container having either a fixed temperature at the bottom (290 K and 310 K test), or zero-flux (300 K test). As one can see, the numerical results closely describe the experimental data in all cases. The good agreement between theory and experiment corroborates the use of the simplified boundary conditions used in the numerical simulations for the far-stream. There is in fact a considerable temperature drop at the interface caused by evaporative cooling that creates thermal gradients on both the liquid and gas side. It is also important to recognize that this drop in interface temperature is strongly dependent on the evaporation rate.

In an isothermal system there exists only positive buoyancy generating an upward flow motion. For the case of an isothermal system with the evaporating surface being a disk, Dollet and Boulonge [18] found a correlation for the Sherwood number consisting of a power law term plus a constant term, \( Sh \approx \frac{G}{\rho} (0.31G^{0.5} + 0.48) \). According to these authors, the two terms result from the difference in local flux contributions along the interface. Once opposing buoyant forces take place, the flow is no longer necessarily upward. For the regimes studied here, the flow is mostly driven downward with the saturated air leaving by the edges. There are also possible stagnation points appearing close to the interface, as shown in Fig. 5. Stagnation points can be a characteristic of natural flows adjacent to horizontal surfaces undergoing opposing thermal-concentration buoyant effects. The numerical results thus suggest that the horizontal and vertical components of the velocity have the same order of magnitude next to the interface (see Fig. 5b) and, for this reason, the boundary layer approach is not applicable [22,23].

The stagnation points appearing next to the interface are associated to flow reversal or recirculation zones. The recirculation zones appear when the flow accelerates downward, creating a flow structure that resembles a toroidal ring-like jet near the rim of the pool. The downward accelerated flow is induced by the sinking motion of air, as a result of the low temperature near the rim. A more detailed analysis shows that the temperature varies along the air-water interface due to the nonuniform radial mass flux [28,29]. Because mass fluxes are higher near the edge, a significant lower temperature drop is expected in those locations. Yet, the resulting temperature of the edge depends on how efficiently heat can be transported to that region, which in turn depends on the thermal resistance of the fluid and of the container [30]. It is possible that in the present study the interface is indeed coldest at the rims, since the thermal conductivity of the very thin-walled (<1 mm) container is lower than the thermal conductivity of liquid water (\( k_{\text{container}} \approx 0.3 – 0.4 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} \)). The colder air near the rim sinks to the bottom or base of the pool, which is at a slightly higher temperature, resulting in the aforementioned local acceleration of the flow. Such flow patterns then sustain the overall downward motion of the air in the near and far flow stream. An example of such ring-like jet structure encircling the recircu-
Recirculation for downward horizontal and vertical flows.

\( \phi = \frac{m_1 - m_{1,e}}{m_{1,s} - m_{1,e}} \)  
\( \theta = \frac{T - T_e}{T_1 - T_e} \)

while masslines and heatlines are respectively given as

\[ \frac{\partial M}{\partial z} = um_1 - \varphi_{12} \frac{\partial m_1}{\partial x} \]
\[ \frac{\partial M}{\partial x} = wm_1 - \varphi_{12} \frac{\partial m_1}{\partial z} \]

and

\[ \frac{\partial H}{\partial z} = \rho c_p w T - k \frac{\partial T}{\partial z} \]  
\[ \frac{\partial H}{\partial x} = \rho c_p w T - k \frac{\partial T}{\partial x} \]

At lower relative humidities, the mass driving force is stronger due to a higher mass concentration difference, and as such more energy is required for evaporation to occur. A more pronounced cooling effect near the air-water interface induces higher thermal gradients, at the same time that it intensifies the sinking motion near the rim. The isolines are then pulled downward around the walls of the pool in consequence of the stronger downward flow. As the relative humidity increases, the flow structure is relatively the same, however less intense and having a more protruding recirculation zone. Notice how, near the air-water interface, the pattern of the masslines and heatlines are more aligned to the shape of the recirculation zone in a wet environment than in a dry environment, indicating that in such condition the mass and heat flows are likely more sensitive to any effect consequent from disturbances of the recirculation zone.

Despite the downward acceleration locally induced near the edge of the air-water interface, which at first suggests that the evaporation rate is dependent on the effective height of the pool, numerical and experimental results show that a universal correlation can be developed to estimate the evaporation of water under downward flow motion, regardless of the geometrical aspect ratio \( H/R \). To evaluate an universal correlation, we conducted experiments with \( H/R \) ratios from 0.6 to 1.7, while the ratio was extended to cover values from 0.3 to 5 in the numerical simulations. By using the form of Eqs. (17a) and (17b) to perform a fit on the
Fig. 7. Isolines for thermal and mass concentration profiles along with the masslines and heatlines for water evaporating into air under downward flow motion. Results are given for RH equal to 10 % and 90 % with a, b, at 300 K. The surface of both the thermal and mass concentration gradients approach mushroom-type shapes, which are deformed under buoyant forcing. Under dry conditions, the high mass flux near the edge induces an intense sinking motion of the air near the rim, that in turn pulls down the isolines around the wall resulting in an enhanced evaporation rate. At high relative humidities (c, d), the flow structure is relatively the same, however with a less intense effect. Notice that, near the air-water interface, the pattern of mass and heat flows resembles more the pattern of the recirculation zones when in a wet environment, indicating that under such condition the mass and heat fluxes at the interface are more susceptible to disturbances of the recirculation zones.

Fig. 8. Sherwood number as a function of the total Grashof number for water evaporating into air under downward flow motion. Experimental results are conducted for Grashof numbers between 50 and 10⁴. The corresponding error bars indicate 2 standard deviations. The numerical results are expanded to cover Grashof numbers from near 1 to 10⁴. The Sherwood number approaches 2 for Grashof numbers less than 10, consistent with the expected in the pure diffusion limit. As the Grashof number increases, the flow velocity also increases, leading the Sherwood number to become proportional to $Gr^{1/4}$ - as it is expected for unbounded natural evaporation. The results take into account different geometrical aspect ratios $H/R$, in turn demonstrating the correlation to be independent of the effective height of the pool. When compared to the correlation proposed by Sparrow et al. (1983) - in which the Sherwood number is proportional to $Gr^{1/4}$, both correlations initially agree at Grashof numbers around 10⁴, but they start to slightly diverge as the Grashof number increases due to the difference in the respective powers. Flow recirculations for $Gr$ numbers below 100 are unstable and can only be achieved numerically under idealized boundary conditions.

Numerical results, we found the following Sh-correlation:

\[
\tilde{Sh} \approx 2 \quad (22a)
\]

for $Gr \leq 10$, and

\[
\tilde{Sh} \approx 1.73 + 0.26 Sc^{1/3} Gr^{1/4} \quad (22b)
\]

for $10 < Gr < 10^5$.

The Sherwood number is represented in Fig. 8 in terms of the absolute value of the total Grashof number, $|Gr_{Re} + Gr_{L}|$. For the sake of simplicity, we suggest for Grashof number values less than 10, the approximation $\tilde{Sh} \approx 2$, which is the value expected in the pure diffusion limit. For $Gr$ values higher than 10, we correlate the Sherwood number to the $1/4$ power on the Grashof number. The experimental values are virtually within $±10\%$ of the nominal correlation result, as seen through the region represented by the shaded area around the suggested curve. The proposed correlation is additionally compared to the one proposed by Sparrow et al. [17], also developed for free evaporation with a downward flow motion and Grashof numbers ranging from $10^4$ to $10^5$. Both correlations initially agree for Grashof numbers around $10^4$, but they slightly diverges as the Grashof number increases, due to the difference in the power on the Grashof number. While Sparrow et al. [17] suggests $Sh \propto Gr^{0.205}$, the correlation proposed here preserves the characteristics of a buoyant laminar flow, besides maintaining the contribution from pure diffusion.

\[
\tilde{Sh} \approx 2 \quad (22a)
\]
The simplified liquid pool approximation (where buoyancy and surface tension are neglected) yields consistent agreement across the range of experimental conditions described here. The good agreement between the numerical simulations and the experiments, particularly with respect to the air-water interface temperature, corroborates the use of the hydrostatic condition in our simulations. This happens because the general downward flow on the air-vapor side is not affected by the simplified liquid pool model as long as the average heat flux from the liquid to the air-liquid interface is correctly estimated. Multiple simulations comparing the solution with buoyancy terms in the liquid pool at different pool bottom temperatures with simulations considering a hydrostatic pool with virtually conduction heat transfer only reveal that the simplified hypothesis results in heat transfer rates to the interface that differ by 2% or less. Nonetheless, the strength and the exact position of the circulation zone highlighted in Fig. 5 may be affected by consideration of other boundary conditions at the bottom of the pool given the weak convective flows in this problem.

The suggested Sh number correlation is also compared to the correlations for upward convection as provided in Fig. 9. In order to obtain all the correlations according to the same characteristic length, the correlations were converted to Sh and Gr number values based on the hydraulic diameter, $D = A/P$, where $A$ is the area of the interface and $P$ is the corresponding perimeter. The correlations corresponding to the upward convection cover in general ranges of $Gr_D$ number of the order of magnitude from $10^4$ to $10^8$. For upward convection, the Sh number is in general given as a power law function where the obtained power $n$ differs in each study, varying from 0.213 for Sharpley and Boelter [14] to 0.306 for Bower and Saylor [12,13]. Such variation is likely to be the result of turbulence effects that appear as the liquid pool is heated and alters the dynamics of water vapor transport away from the interface [12,13].

The relative contribution from the heat fluxes present in the energy balance at the interface (Eq. 12g) to the total energy of vaporization is nearly independent of environmental conditions. It does, however, depend on the thermal condition of the container, as shown in Fig. 10. When the bottom of the container is fixed at the temperature $T_0$, the heat flux from the liquid accounts for approximately 80% of the total required energy, while the heat flux from the gas side and from surface radiation are responsible for relative amounts close to 10% each. Consequently, the heat flux from the liquid is the dominating term. As shown by the results from the three different environmental conditions presented (290 K, 300 K and 310 K, all at 1 atm), the relative contributions to the total energy of vaporization stays virtually the same, regardless of the environmental condition. When the thermal condition at the bottom of the container changes to zero flux, the heat flux from the liquid remains as the leading term, but now having a reduced relative contribution: the heat flux from the liquid in this case accounts for approximately 60% of the total energy, as the contribution from the gas and from surface radiation increase to 15% and 25% re-

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**Fig. 9.** Comparison among several Sh number correlations for downward (straight lines) and upward (dashed lines) convection. The correlations obtained from previous studies were converted to the same characteristic length (hydraulic diameter) for comparison. For upward convection correlations the power-law exponent $n$ may differ due to turbulence effects and air-water interface conditions that depend on the heat flow input to the liquid pool.

**Fig. 10.** Heat flux contribution from numerical results for water evaporating into air under downward flow motion. Each term in the energy balance (see Eq. 12g) is evaluated by varying the RH, at three different ambient conditions: 290 K (blue), 300 K (gray) and 310 K (orange) - all at 1 atm. The relative contributions are influenced by the thermal condition of the container, which in this study was analysed by changing the thermal condition at the bottom. For instance, when the temperature of the bottom is fixed at the ambient temperature (condition a), the heat flux from the liquid $q_{lw.}$ accounts for approximately 80% of the total heat flux, while the heat flux from the gas $q_{w.}$ and from radiation $q_{r.}$ share similar parcels of 10% each. When the condition of the bottom becomes zero flux (condition b), $q_{lw.}$ reduces to approximately 60%, while $q_{r.}$ increases to around 25% and $q_{w.}$ to 15%. Overall, the relative contribution from each transport mechanism is virtually the same regardless of environmental condition. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
spectively. As expected, the heat fluxes coming from the gas side and from radiation are enhanced as the energy input through the bottom of the container is lowered.

7. Concluding remarks

In this work we investigate the evaporation of water from a circular pool with opposing buoyant and downward thermal flows at near ambient conditions. The main objective is to extend previous studies to much lower Grashof (or Rayleigh) number values, and to examine the effects of recirculation zones on the evaporation rates. The governing equations are solved numerically and validated against experimental results obtained under highly controlled conditions. The very strong agreement between experiments and theory validate the choices for the radiative-convective energy balances at the interface. Evaporative cooling effects result in thermal gradients on both the gas and liquid sides, and these small gradients generate flow patterns that developed into stable configurations for Gr numbers greater than 50. For free evaporation with dominant downward motion, the heat flux from the liquid provides the most significant contribution to the total heat of vaporization, although the relative contribution of each transport mechanism depends on the energy supplied to the interface through the bottom of the pool (see Fig. 10). We show that the vertical and horizontal components of the velocity near the air-water interface are of same order of magnitude, which precludes the use of simplifying boundary layer approximations in the study of weakly convective flow at low Grashof numbers. We also demonstrate that the thermal and mass concentration isosurfaces are spherically-symmetric, sensitive to disturbances caused by the surrounding buoyant convective forcing: because of the drop in temperature near the rim of the pool, a sinking motion of the air induces a local acceleration of the flow near the rim that encircles recirculations zones originating at the air-water interface. When in an wet environment with minimized external disturbances (e.g., numerical simulations using a uniform boundary condition for temperature uniformity), the evaporation yields a Sherwood number based on the radius of the pool only slightly higher than 2 for Grashof number less than 10, which is the value expected in the pure diffusion limit. For Grashof number values higher than 10 and up to $10^5$, a Sh-correlation is proposed: $Sh \approx 1.73 + 0.26 Sc^{1/3}Gr^{1/4}$ for $10 \leq Gr < 10^5$. The proposed correlation covers geometrical aspect ratios $H/R$ from 0.3 to 5, while obeying the scaling law associated with unbounded laminar buoyant flows.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Coefficient of volumetric expansion

The thermal and mass coefficients of volumetric expansion, $\beta_T$ and $\beta_m$, are expressed for a binary mixture as

$$\beta_T = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_{\rho, \xi} \approx \frac{1}{T_e},$$  

(A.1)

$$\beta_m = \rho \left( -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial \rho \ln \xi} \right)_{\rho, \xi} \right) \approx \frac{M_2}{M_1} - 1,$$  

(A.2)

with $M$ being the molecular weight of the species in question. Note that Eq. (A.2) implies $\rho_2 \gg \rho_1$ and $x_2 \gg x_1$. The former is approximately true for water vapor (species 1) in air (species 2), while the latter is always true for the same mixture under tropospheric conditions.

Appendix B. Saturation pressure of water vapor

The saturated vapor pressure in this work is estimated using the Tetens equation [25]

$$P_{sat} = 6.01078 \exp \left( \frac{17.27 T}{T + 234.5} \right),$$  

(B.1)

where $T$ is in degrees celsius and $P_{sat}$ is in kilopascals.

Appendix C. Binary diffusion coefficient for water vapor in air

The binary diffusion coefficient of water vapor in air used in this study follows the empirical relation [24]

$$D_{12} = 1.97 \times 10^{-5} \frac{(P_1)^{1/3}}{T_0^{1/2}} \frac{T^2}{T_0^2},$$  

(C.1)

where $P_0 = 1$ atm, $T_0 = 256$ K and $D_{12}$ given in m$^2$/s.

References


